Chapter 4
Multidimensional Flow Analysis

4-1. Introduction

a. Definitions. Multidimensional flow analysis is the description and/or prediction of the detailed hydraulic characteristics of a particular flow situation in more than one dimension (direction). "Hydraulic characteristics" refers to the following properties of the flow, discharge, velocity, water surface elevation (depth), boundary shear stress, rate of energy dissipation, and constituent or sediment transport rate. "Particular flow situation" refers to the specific body of water, location therein, physical setting, alternative design configurations, and flows (steady or dynamic) to be studied.

b. Description. This type of analysis recognizes velocity and depth variations in either two or three directions. For example, flow patterns in an estuary or at a river confluence may exhibit significant velocities in both the streamwise and transverse directions. A one-dimensional flow model does not explicitly consider these transverse effects. Horizontal, depth-averaged, two-dimensional flow models such as RMA-2 (King 1988, Gee et al. 1990) are used in river hydraulics studies mainly for two purposes: (1) to analyze two-dimensional flow patterns in detail at some area of interest (such as at bridge crossings, the confluence of two channels, flow around islands, etc.) or (2) to analyze the flow behavior on an unbounded alluvial fan or in a wide river valley. Two- and three-dimensional models can be used for both steady and unsteady flow conditions. Sediment transport and water quality analyses can also be done with multidimensional flow models such as TABS-2 (Thomas and McAnally 1985). TABS-2 has primarily been used for simulating the sedimentation processes in reservoirs, estuaries, and complex river channels.

c. Techniques. The techniques discussed in this and the following two chapters are strictly applicable only for rigid boundary (bed and banks) situations. Techniques that are used for movable boundary problems (Chapter 7) are extensions of the techniques presented in Chapters 4 through 6. In selecting an appropriate technique, or suite of techniques, the engineer must identify the important physical processes that need to be recognized in the analysis. Resources and data necessary to manage and perform the appropriate level of analysis need to be identified early in the study plan (refer to Chapter 3).

4-2. Limitations of One-Dimensional Analysis

Flow in a channel or river is quite often viewed as being one-dimensional in the streamwise direction. This means that the stage (water surface elevation), velocity, and discharge vary only in the streamwise direction. Subdivision of cross sections, however, provides an approximate method of accounting for transverse roughness and velocity distributions. This approach provides a simplified mathematical description of the flow for water surface elevation prediction (see Chapters 5 and 6). More detailed analysis of flow velocities and directions requires representation of the flow physics (conservation of mass and momentum) in two and, sometimes, three dimensions. The engineer should understand the capabilities, limitations, and effort required to perform the various levels of analysis described in this and the following chapters. This information should be used to make an informed decision regarding the technical approach needed to meet the study objectives and to define the resources necessary to manage and perform the study.

4-3. Equations of Flow

The principles of mass and momentum conservation are presented below in generalized three-dimensional form. Simplifying assumptions allow the reduction of the equations to two dimensions and to one dimension.

a. Conservation of momentum. The conservation of momentum equations in the x (horizontal), y (horizontal), and z (vertical) directions are respectively:

$$\begin{aligned}\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} + \rho w\frac{\partial u}{\partial z} \\ - \frac{\partial}{\partial x}(\varepsilon_{xx}\frac{\partial u}{\partial x}) - \frac{\partial}{\partial y}(\varepsilon_{xy}\frac{\partial u}{\partial y}) \\ - \frac{\partial}{\partial z}(\varepsilon_{xz}\frac{\partial u}{\partial z}) - \frac{\partial p}{\partial x} - \tau_x = 0\end{aligned} \tag{4-1}$$

$$\begin{aligned}\rho\frac{\partial v}{\partial t} + \rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} + \rho w\frac{\partial v}{\partial z} \\ - \frac{\partial}{\partial x}(\varepsilon_{yx}\frac{\partial v}{\partial x}) - \frac{\partial}{\partial y}(\varepsilon_{yy}\frac{\partial v}{\partial y}) \\ - \frac{\partial}{\partial z}(\varepsilon_{yz}\frac{\partial v}{\partial z}) - \frac{\partial p}{\partial y} - \tau_y = 0\end{aligned} \tag{4-2}$$

$$\rho\frac{\partial w}{\partial t} + \rho u\frac{\partial w}{\partial x} + \rho v\frac{\partial w}{\partial y} + \rho w\frac{\partial w}{\partial z} - \frac{\partial}{\partial x}(\varepsilon_{zx}\frac{\partial w}{\partial x}) - \frac{\partial}{\partial y}(\varepsilon_{zy}\frac{\partial w}{\partial y}) - \frac{\partial}{\partial z}(\varepsilon_{zz}\frac{\partial w}{\partial z}) - \frac{\partial p}{\partial z} - \rho g - \tau_z = 0 \tag{4-3}$$

b. Conservation of mass. The conservation of mass equation is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4-4}$$

where

x,y,z = the Cartesian coordinate directions.
u,v,w = velocity components in the x,y,z directions, respectively.
t = time.
g = the acceleration due to gravity.
p = pressure.
ρ = fluid density.[1]
ε_{xx}, ε_{xy}, etc. = the turbulent exchange coefficients which describe the diffusion of momentum in the direction of the first subscript to that of the second subscript.
τ_x, τ_y, τ_z = terms representing the influence of boundary shear stresses.

4-4. Significance of Terms

a. Accelerations. The terms in these equations represent forces (e.g., the pressure gradient $\partial p/\partial x$), local (temporal) accelerations (e.g., $\partial u/\partial t$), convective accelerations (e.g., $u\partial u/\partial x$), and mass continuity. The momentum equations are derived by application of Newton's Second Law of Motion. The basic assumptions made are that the fluid is incompressible (constant density) and that the effects of turbulent momentum exchange can be simulated with an "eddy viscosity" (Boussinesq assumption). A rigorous derivation of these equations may be found in Rouse (1938) and French (1985).

b. Forces. The forces in Equations 4-1 to 4-3 are those of gravity, pressure, boundary friction, and exchange of momentum due to turbulence. Some formulations of these equations may also include forces due to wind, ice, and the earth's rotation. For most riverine situations, wind and the earth's rotation (Coriolis effect) are not important; they may become important for bodies of water with length scales of tens of miles, and may become dominant for large bodies of water such as the Great Lakes. The continuity equation (4-4) represents an accounting of water mass of constant density. Other formulations of these equations, such as used in estuaries, oceans, and lakes may include variable density.

4-5. Use of Equations of Flow

a. General. Equations 4-1 to 4-4 are applicable to all river and channel flow situations that satisfy the assumptions of constant density and a rigid (or at least slowly changing) boundary. The difficulty lies in solving the equations. The only reliable and routinely used engineering tool for solving the three-dimensional equations at this time (1991) is the physical model. Numerical models (computer programs), however, are routinely and successfully used for solving the two- and one-dimensional simplifications of the above equations. Three-dimensional numerical models are presently under development and undergoing field testing with some applications being reported. A major study of Chesapeake Bay using a three-dimensional numerical model is reported by Kim et al. (1990) and Johnson et al. (1991).

b. Traditional approaches. "Traditional" approaches to river hydraulics studies separate continuity, or storage, routing HEC-1, (U.S. Army Corps of Engineers 1990a) to determine the discharge, from the one-dimensional steady flow computations HEC-2, U.S. Army Corps of Engineers 1990b) used to determine water surface elevations. Application of Equations 4-1 to 4-4 achieves the combined result of both routing and water surface elevation computation in a single computation. The "traditional" techniques presented in Chapters 5 and 6 are based on simplifications of, or approximations to, the equations presented above. There are many river analysis problems that can be satisfactorily evaluated with simplified methods. The focus of this chapter, however, is the analysis of more complex hydraulics problems in greater detail and resolution than is available with the traditional techniques.

4-6. Two-Dimensional Flow Conditions

a. General. For many rivers the width to depth ratio is 20 or more. In these cases, and for many common

[1] In general, density is a function of temperature, salinity, and pressure and is described with an additional "equation of state", see Sverdurp et al. (1942) and Wiegel (1964).

applications, the velocity variations in the vertical are much less important than those in the transverse and streamwise directions. The above equations can be averaged in the vertical (i.e., depth averaged) to yield the two-dimensional equations for flow in the horizontal plane which adequately describe the flow field for most rivers with these characteristics. Two-dimensional flow analysis should be considered for river hydraulics problems where the direction or distribution of flow is of importance, either directly or because it affects variables of interest such as water surface elevation, and cannot be assumed as is required by a one-dimensional analysis. Figure 4-1 depicts a situation where the flow could be adequately modeled by a two-dimensional approach. Figure 4-2 contrasts the one-dimensional approach to the same problem where one must select cross sections perpendicular to the flow direction. While it may be possible to calibrate a one-dimensional model to reproduce the overall energy loss in this flow field, key components of the flow field such as flow separations and recirculation zones would not be reproduced at all by a one-dimensional model.

b. Specific situations. Another situation that may require a two-dimensional analysis is that of a bridge with multiple openings crossing a broad, flat, floodplain. In this case the water surface elevation upstream of the bridge may be strongly dependent upon the distribution of flow among the bridge openings. This distribution of flow cannot be directly computed with a one-dimensional approach. Such situations require that the engineer carefully select the level of analysis; physical model, numerical model, or other analytical technique (refer to Chapter 3).

c. Dynamic simulations. Multidimensional flow analysis can be either unsteady (dynamic) or steady. Dynamic simulations require substantially more

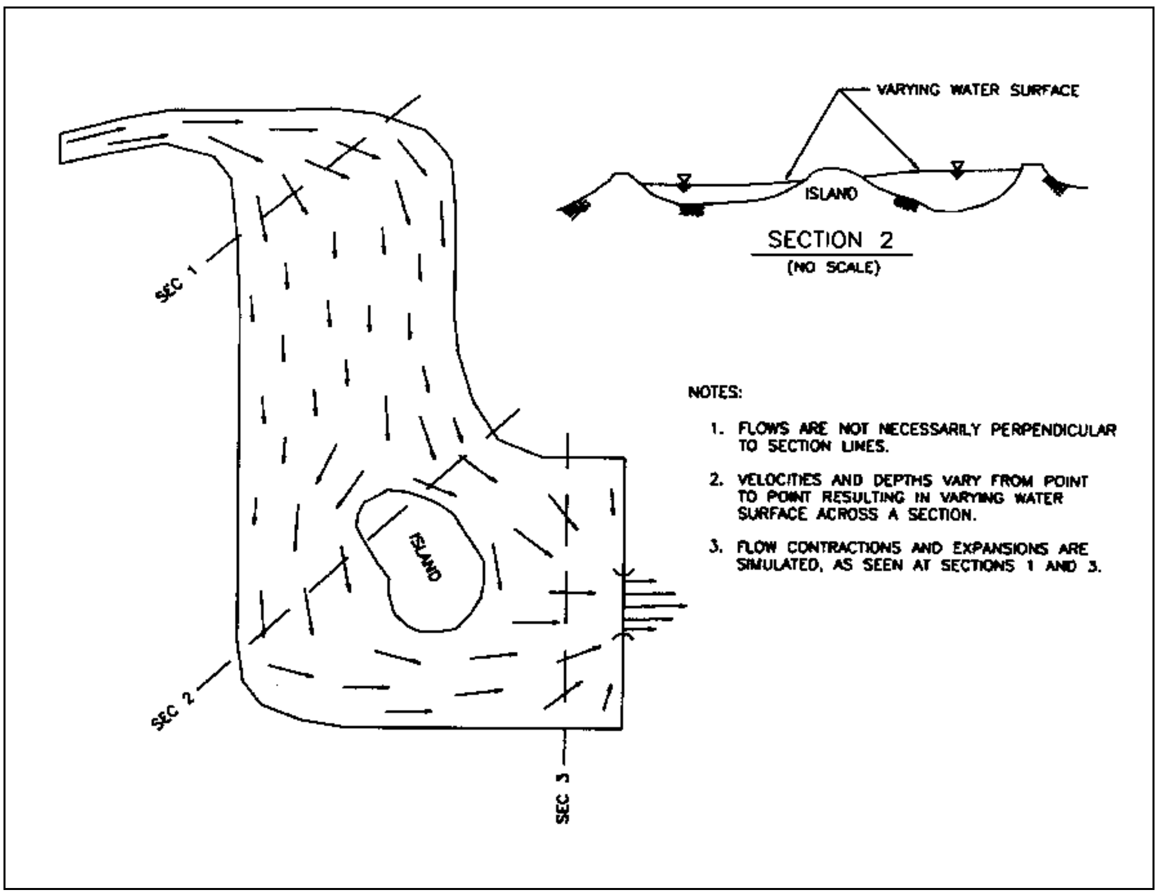

Figure 4-1. Two-dimensional flow representation in cache creek settling basin

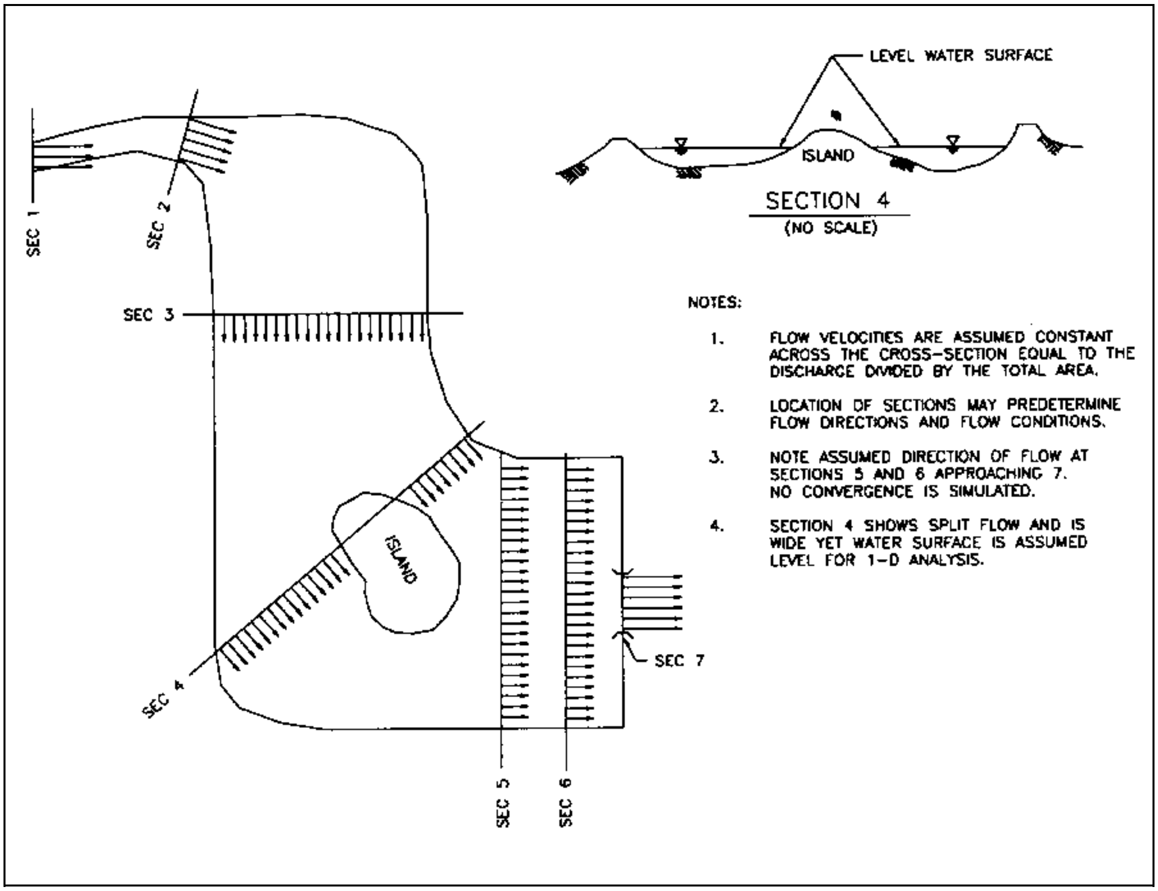

Figure 4-2. One-dimensional flow limitation in cache creek settling basin

computational effort than steady state simulations (Gee et al. 1990). Furthermore, the analysis and presentation of results from a dynamic simulation is much more complex than that of a steady flow simulation. Therefore, in designing a multidimensional flow study it is important to decide whether a dynamic analysis is necessary. In most riverine studies, steady flow is adequate; in tidal systems it never is. The alternative design configurations and/or flows to be studied must be carefully selected to maximize study efficiency and ensure that all relevant situations are analyzed. Refer to Appendix C for more detailed information regarding the contents of a work plan for the application of a multidimensional flow model.

4-7. Available Computer Programs

a. Use. Use of two-dimensional numerical modeling techniques is becoming a routine and accepted engineering practice. Inexperienced analysts should seek guidance and advice from experienced engineers, particularly early in the study, to define data and resources needed for complex model applications. Application of such a sophisticated numerical flow model for a one-time study may best be accomplished with the assistance of a Corps laboratory or outside contractor. Development of in-house expertise for such applications, while requiring significant initial investment of resources in training, may result in future savings if several similar studies are planned. Consideration must be given to model availability (public versus proprietary), applications experience, training and documentation, features, applicability, and required computer resources. Good graphics capabilities, both screen and color hardcopy, are essential to perform efficient and successful applications of multidimensional flow models. Multidimensional flow model applications should be integrated with CADD and/or GIS as appropriate for study needs.

b. RMA-2. Computer programs are readily available for conducting two-dimensional river hydraulics analyses in the horizontal plane (Thomas & McAnally 1985, U.S. Department of Transporation 1989). Commonly used in the Corps of Engineers is RMA-2 (King 1988) which is the hydraulics module of the TABS-2 modeling system (Thomas and McAnally 1985). Synopses of these and other programs are presented in HEC (U.S. Army Corps of Engineers 1982b). RMA-2 solves the vertically (i.e., depth) averaged version of equations 4-1 to 4-4; written as shown below.

Momentum equations:

$$h\frac{\partial u}{\partial t} + uh\frac{\partial u}{\partial x} + vh\frac{\partial u}{\partial y} + gh\frac{\partial a}{\partial x} + gh\frac{\partial h}{\partial x} - \frac{h\varepsilon_{xx}}{\rho}\frac{\partial^2 u}{\partial x^2} - \frac{h\varepsilon_{xy}}{\rho}\frac{\partial^2 u}{\partial y^2} + S_{fx} + \tau_x = 0 \tag{4-5}$$

$$h\frac{\partial v}{\partial t} + uh\frac{\partial v}{\partial x} + vh\frac{\partial v}{\partial y} + gh\frac{\partial a}{\partial y} + gh\frac{\partial h}{\partial y} - \frac{h\varepsilon_{yx}}{\rho}\frac{\partial^2 v}{\partial x^2} - \frac{h\varepsilon_{yy}}{\rho}\frac{\partial^2 v}{\partial y^2} + S_{fy} + \tau_y = 0 \tag{4-6}$$

Continuity equation:

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0 \tag{4-7}$$

where

x,y = the horizontal coordinate directions.
u,v = velocity components in the x and y directions, respectively.
t = time.
g = the acceleration due to gravity.
a = the bottom elevation.
h = the depth.
ρ = fluid density.
ε_{xx}, ε_{xy}, etc. = the turbulent exchange coefficients which describe the diffusion of momentum in the direction of the first subscript to that of the second subscript.
S_{fx}, S_{fy} = terms for the nonlinear Manning or Chezy representation of bottom friction.
τx, τy = terms representing boundary shear stresses other than bottom friction (e.g., wind), these terms also include the Coriolis effect.

4-8. Data Requirements

It is useful to think of "data" in three categories: analysis input data, calibration data, and validation or confirmation data. These categories are useful when identifying data requirements for both physical and numerical models.

a. Analysis input data. Analysis input data are those items required to operate the model. They consist of a geometric description of the study area (e.g., cross sections in one-dimension, contour maps, or a digital terrain model for two-dimensions), flow to be analyzed (a single discharge for steady flow, or a hydrograph for unsteady flow), other boundary conditions such as stages or rating curves, and various coefficients that approximate the effects of friction and turbulence. Of these, the geometric description of the study area is usually the most time consuming to develop and schematize; it is, however, not necessarily the most important data in terms of simulation accuracy (U.S. Army Corps of Engineers 1986). The density (i.e. resolution) and accuracy required of the flow and geometric data are governed, fundamentally, by the study purpose, not the analysis technique (Cunge et al. 1980).

b. Calibration data. Calibration data consist of field observations that are used to evaluate the performance of a model and adjust the coefficients to improve its performance, if necessary. "Performance" is a qualitative, or subjective, measure of the degree to which the model faithfully reproduces the field observations. This measure is applied by the engineer performing the study and documented by means of the reporting process. The complexities of river hydraulics do not allow the setting of objective criteria to measure the accuracy of calibration. Whether the model's performance is acceptable depends on study objectives, sensitivity of study outcomes to model results, and reliability of field data.

(1) The weight given to the performance of a model with regard to different hydraulic variables, such as water surface elevation or velocity, will vary with study objectives, data availability and reliability, and the judgment of the engineer. For example, floodway studies focus on accurate computation of the water surface elevation while constituent transport studies require accurate reproduction of velocity, water discharge, and mixing. Surrogate data should be used with caution. For example, if the study objectives require the prediction of discharge, prototype discharge should be measured for calibration rather than derived from a rating curve.

(2) In the context of two-dimensional modeling for river hydraulics, the study objectives usually require the prediction of velocity or stage. Field measurements of velocity must include the direction as well as the magnitude. Most two-dimensional models used for river hydraulics compute vertically averaged velocities; therefore, the field data must be converted to vertical averages for proper model-prototype comparisons. For most situations, it is adequate to use the average of the velocities measured at 0.2*depth and 0.8*depth (French 1985). Depth must also be obtained at the locations of the velocity measurements. "Depth" alone is of limited value; one should also have the corresponding water surface or bed surface elevation. Similarly, to calibrate a model for stage prediction, one should have field measurements of stage and the variation of stage with time at many locations within the study area. Also, the discharge(s) at the time(s) of those measurements must be known.

c. Validation data. Validation data are field observations not used in calibration that are used to provide an independent check on model performance (ASCE 1982). The above considerations for calibration data also apply to validation data.

4-9. Data Development and Model Calibration

a. Geometry. An accurate geometric description of the flow region is a primary requirement. "Accurate" here means that the key flow controlling and conveying features of the study area are appropriately represented in the field data. The engineer should be aware of the origin and veracity of the field data. Ideally, the area of interest is described by a detailed digital terrain model or contour map of adequate resolution for the study needs. Refer to EM 1110-2-1003 and "Accuracy of Computed Water Surface Profiles" (1986). Most existing model data are, however, in the format of cross sections (HEC-2). Direct use of HEC-2 style data for two-dimensional or one-dimensional unsteady simulations should be tempered by the following considerations: (1) the HEC-2 cross sections may not have been chosen to best represent the direction and distribution of flow, (2) off-channel storage areas (important for dynamic simulations) may have been neglected when surveying the cross sections, and (3) the sections may not be appropriate for the objectives of the present study. Therefore, before using an existing HEC-2 (or other one-dimensional steady flow) data set, thoroughly check the data for conformance with the needs of the present study objectives. The use of cross sections to develop two-dimensional model input requires that the sections be registered (located) on a topographic map or aerial photograph and the contours filled in, usually by hand.

b. Bottom roughness. In most two-dimensional riverine situations, bottom roughness can be described in the same fashion as would be used for a "traditional" one-dimensional (HEC-2) analysis (refer to Chapter 6). Due to the ability of the two-dimensional approach to incorporate spatial variation of roughness, aerial photographs or topographic maps can be used to identify regions of uniform roughness, such as clumps of vegetation, changes in bed material or bed forms. As in the one-dimensional approach, the roughness coefficients selected from field inspection (which is essential for successful modeling) will probably need to be modified in the calibration process. Should the calibration process indicate the need for values of coefficients that are outside the range suggested by good engineering judgment, one should closely inspect the geometric data, flow data, boundary condition specifications, and calibration data. Most often it is flawed geometric data, or the manner in which it is interpreted by the engineer and used by the numerical model that is the cause of a poor simulation.

c. Turbulent exchange coefficients. Two-dimensional flow models require turbulent exchange coefficients, often called eddy diffusivities, which represent the internal shear forces created by the transfer of momentum between faster and slower regions of flow by means of turbulent mixing. This can actually be observed in most rivers by watching surface boils and eddies move about in the flow. These coefficients reflect, somewhat, the energy losses that are described by the expansion and contraction coefficients in one-dimensional models. The values of these coefficients cannot be directly measured nor observed. Calibrated expansion-contraction coefficients cannot be directly translated into values for the turbulent exchange coefficients. Guidance on selection of values for the turbulent exchange coefficients is provided in the documentation for two-dimensional models (e.g., TABS-2, Thomas and McAnally 1985). These coefficients primarily effect velocity distributions and should be calibrated based on velocity distributions measured in the field. If measurements are not available, information from photographs (both ground and aerial) of the flow or sketches of observed flow patterns can be of use. Some flow situations such as a jet entering a still body of water are momentum dominated. In these cases, the exchange coefficients are very important. Most open river problems are friction dominated, however, and the model results may not be very sensitive to the value selected for the turbulent exchange coefficients. A general approach is to first calibrate the roughness coefficients (Manning's *n* values) to reproduce the energy loss or water surface gradient through the study reach and then adjust the turbulent exchange coefficients to match the observed or expected velocity distribution. The exchange coefficients should be set to the high end of the expected range first, then lowered until the desired velocity pattern is reproduced by the model. In general, the higher the coefficients, the more uniform the velocity distribution; the lower the coefficients, the more readily does flow separation and eddy formation take place. Two-dimensional models (as with one-dimensional models) should be calibrated to steady flow conditions first, if possible, before attempting calibration to an unsteady flow event (Cunge et al. 1980).

d. Field data. In addition to thoroughly inspecting the study area, the analyst should be familiar with the manner in which field observations are made, that is, the type of instruments used and the conditions under which the data were obtained. Data reduction techniques may also affect the accuracy and variability of the observations. The analyst should not consider field data to be perfectly accurate nor necessarily representative of field conditions over the complete range of circumstances to be studied. Internal consistency of field data should be checked if at all possible. For example, when using velocity observations for calibration of a two-dimensional model in steady flow conditions, one should calculate the discharge from the velocity and depth measurements and compare it to the discharge obtained from a nearby gage at the same time as the velocity measurements were made.

4-10. Example Applications

Most applications of two-dimensional horizontal models to date have been in estuarial environments; some of these applications are presented in "Two-Dimensional Flow Modeling" (U.S. Army Corps of Engineers 1982b), McAnally et al. (1984a, 1984b), and MacArthur et al. (1987). A recent study that evaluated the effects of deepening a ship channel on velocity patterns and shoaling is discussed by Lin and Martin (1989). Computation of velocity distributions in a river downstream from a hydropower project is presented in Gee and Wilcox (1985). Impacts of highway bridge crossings on water surface elevations are discussed in Lee (1980), Tseng (1975), and Heltzel (1988). Effects of dikes on the flow distribution in a river was investigated using TABS-2 by Thomas and Heath (1983). Use of two-dimensional modeling to analyze effects on river stage of a major channel encroachment is presented in Stewart et al. (1985). In this study use of a one-dimensional model did not produce credible results because values of the expansion-contraction coefficients governed the outcome and, as this was a design study, there were no field data for their calibration. Results were much less sensitive to the values of the turbulent exchange coefficients because the major flow patterns and separation areas were calculated directly by the two-dimensional model. It is the effects (energy losses) of these separation areas that the expansion-contraction coefficients attempt to describe. Use of RMA-2 to model flood movement in a large river channel-floodplain system is presented in Gee et al. (1990). This paper also describes the computational resources required to perform such a study. Use of a two-dimensional model to analyze distribution of flow in the St. Lawrence River is documented by Heath (1989).